<u>AMENDMENTS TO THE CLAIMS</u>

This listing of claims will replace all prior versions, and listings, of claims in the application:

<u>Listing of Claims:</u>

1. (Currently amended) A method for using interval techniques within a computer system to solve a multi-objective optimization problem, comprising:

receiving a representation of multiple objective functions $(f_1, \ldots, f_n)$ at the computer system, wherein $(f_1, \ldots, f_n)$ are scalar functions of a vector $\mathbf{x} = (x_1, \ldots, x_n)$;

receiving a representation of a domain of interest for the multiple objective functions;

storing the representations in a memory within the computer system; and

performing an interval optimization process to compute guaranteed bounds on a Pareto front for the objective functions $(f_1, \ldots, f_n)$, wherein for each point on the Pareto front, an improvement in one objective function cannot be made without adversely affecting at least one other objective function;

wherein performing the interval optimization process involves applying a direct-comparison technique between subdomains of the domain of interest to eliminate subdomains that are certainly dominated by other subdomains<u>,</u>

<u>wherein performing the interval optimization process involves applying a gradient technique to eliminate subdomains that do not contain a local Pareto optimum,</u>

wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an intersection of certainly negative gradient regions $\mathbf{C}_j$ for each objective function f_j is non-empty, $\bigcap_{j=1}^{n} \mathbf{C}_j([\mathbf{x}]_i) \neq \emptyset$, and

wherein the certainly negative gradient region $\mathbf{C}_j$ for objective function f_j is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$).

2. (Cancelled)

3. (Cancelled)

4. (Currently amended) The method of ~~claim 3~~claim 1, wherein the method further comprises iteratively:

bisecting remaining subdomains that have not been eliminated by the gradient technique; and

applying the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

5. (Original) The method of claim 4, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions (f_1, ..., f_n) over the subdomain.

6. (Original) The method of claim 4, wherein the direct-comparison technique is applied once for every n iterations of the gradient technique.

7. (Original) The method of claim 6, wherein the iterations continue until either a predetermined maximum number of iterations are performed, or the largest area of any subdomain is below a predetermined value.

8. (Original) The method of claim 1,

wherein a subdomain **U** certainly dominates a subdomain **V** if every point $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and

wherein a point **u** dominates a point **v** under minimization if,

$$u_i \# v_i, i = 1, \ldots, n, \text{ and}$$

$$u_i < v_i \text{ for some } i \in \{1, \ldots, n\}.$$

9. (Currently amended) A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for using interval techniques within a computer system to solve a multi-objective optimization problem, wherein the computer-readable storage medium can be any device that can store code and/or data for use by a computer system, the method comprising:

receiving a representation of multiple objective functions $(f_1, \ldots, f_n)$ at the computer system, wherein $(f_1, \ldots, f_n)$ are scalar functions of a vector $\mathbf{x} = (x_1, \ldots, x_n)$;

receiving a representation of a domain of interest for the multiple objective functions;

storing the representations in a memory within the computer system; and

performing an interval optimization process to compute guaranteed bounds on a Pareto front for the objective functions $(f_1, \ldots, f_n)$, wherein for each point on the Pareto front, an improvement in one objective function cannot be made without adversely affecting at least one other objective function;

wherein performing the interval optimization process involves applying a direct-comparison technique between subdomains of the domain of interest to eliminate subdomains that are certainly dominated by other subdomains,

wherein performing the interval optimization process involves applying a gradient technique to eliminate subdomains that do not contain a local Pareto optimum,

wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an intersection of certainly negative gradient regions $\mathbf{C}_j$ for each objective function f_j is non-empty, $\bigcap_{j=1}^{n} \mathbf{C}_j([\mathbf{x}]_i) \neq \varnothing$, and

wherein the certainly negative gradient region $\mathbf{C}_j$ for objective function f_j is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$)..

10. (Cancelled)

11. (Cancelled)

12. (Currently amended) The computer-readable storage medium of ~~claim 11~~claim 9, wherein the method further comprises iteratively:

bisecting remaining subdomains that have not been eliminated by the gradient technique; and

applying the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

13. (Original) The computer-readable storage medium of claim 12, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions (f_1 , ..., f_n) over the subdomain.

14. (Original) The computer-readable storage medium of claim 12, wherein the direct-comparison technique is applied once for every n iterations of the gradient technique.

15. (Original) The computer-readable storage medium of claim 14, wherein the iterations continue until either a predetermined maximum number of iterations are performed, or the largest area of any subdomain is below a predetermined value.

16. (Original) The computer-readable storage medium of claim 9,

wherein a subdomain **U** certainly dominates a subdomain **V** if every point $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and

wherein a point **u** dominates a point **v** under minimization if,

$$u_i \# v_i,\ i = 1, \ldots, n, \text{ and}$$

$$u_i < v_i \text{ for some } i \in \{1, \ldots, \mathrm{n}\}.$$

17. (Currently amended) An apparatus that uses interval techniques to solve a multi-objective optimization problem, comprising:

a receiving mechanism configured to receive a representation of multiple objective functions (f_1 , ..., f_n), wherein (f_1 , ..., f_n) are scalar functions of a vector $\mathbf{x} = (x_1, \ldots, x_n)$;

wherein the receiving mechanism is configured to receive a representation of a domain of interest for the multiple objective functions;

a memory configured to store the representations; and

an interval optimizer configured to performing an interval optimization process to compute guaranteed bounds on a Pareto front for the objective functions ($f_1, \ldots, f_n$), wherein for each point on the Pareto front, an improvement in one objective function cannot be made without adversely affecting at least one other objective function;

wherein the interval optimizer is configured to apply a direct-comparison technique between subdomains of the domain of interest to eliminate subdomains that are certainly dominated by other subdomains,

wherein the interval optimizer is configured to apply a gradient technique to eliminate subdomains that do not contain a local Pareto optimum,

wherein a subdomain $[\mathbf{x}]_i$ is eliminated by the gradient technique if an intersection of certainly negative gradient regions $\mathbf{C}_j$ for each objective function f_j is non-empty, $\bigcap_{j=1}^{n} \mathbf{C}_j([\mathbf{x}]_i) \neq \emptyset$, and

wherein the certainly negative gradient region $\mathbf{C}_j$ for objective function f_j is the intersection of $\underline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the minimum angle $\underline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$) and $\overline{\mathbf{N}}_j([\mathbf{x}]_i)$ (the negative gradient region associated with the maximum angle $\overline{\theta}_j$ of the gradient of f_j over the subdomain $[\mathbf{x}]_i$).

18. (Cancelled)

19. (Cancelled)

20. (Currently amended) The apparatus of ~~claim 19~~claim 17, wherein the interval optimizer is configured to iteratively:

bisect remaining subdomains that have not been eliminated by the gradient technique; and to

apply the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

21. (Original) The apparatus of claim 20, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions (f_1 , …, f_n) over the subdomain.

22. (Original) The apparatus of claim 20, wherein the direct-comparison technique is applied once for every *n* iterations of the gradient technique.

23. (Original) The apparatus of claim 22, wherein the iterations continue until either a predetermined maximum number of iterations are performed, or the largest area of any subdomain is below a predetermined value.

24. (Original) The apparatus of claim 17,

wherein a subdomain **U** certainly dominates a subdomain **V** if every point $\mathbf{u} \in \mathbf{U}$ dominates every point $\mathbf{v} \in \mathbf{V}$; and

wherein a point **u** dominates a point **v** under minimization if,

$$u_i \# v_i,\ i = 1, \ldots, n, \text{ and}$$

$u_i < v_i$ for some $i \in \{1, \ldots, n\}$.